Sem 2 Review Calc OK

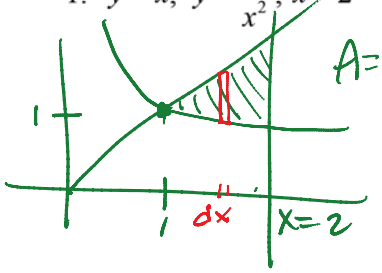
Wednesday, May 20, 2015
7:52 AM

AP Calculus AB
2nd semester review
Calculator OK

Name_____

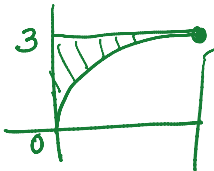
Find the area enclosed by the lines and curves. You must show your set up, can solve using a calculator.

1. $y = x,\ y = \frac{1}{x^2},\ x = 2$



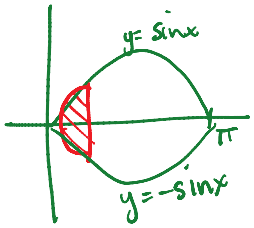
$A = \int_1^2 \left[x - \frac{1}{x^2}\right] dx$

2. $x = 2y^2,\ x = 0,\ y = 3$



$\boxed{\int_0^3 2y^2\,dy}$

3. The base of a solid is the region enclosed between the graphs of $y = \sin x$ *and* $y = -\sin x$ from x=0 to $x = \pi$. Each cross section is perpendicular to the x-axis is a semi-circle with diameter connecting the two graphs. Find the volume of the solid.



$d = 2\sin x$

$r = \sin x$

$A = \frac{1}{2}\pi \sin^2 x$

$\boxed{V = \int_0^{\pi} \frac{1}{2}\pi \sin^2 x\,dx}$

4. Find the volume of the solid generated by revolving the region enclosed by the parabola $y^2 = 4x$ and the line y = x about:

$y = \sqrt{4x}$ $\quad x = \frac{y^2}{4}$

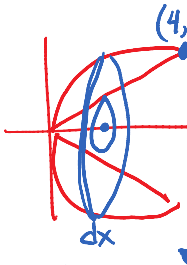
a. the x-axis

b. the y-axis

c. the line x=4

d. the line y = 4

(a)

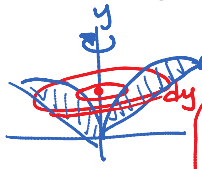


$r = x$

$R = \sqrt{4x}$

$\boxed{V = \pi \int_0^4 \left((\sqrt{4x})^2 - x^2\right) dx}$

(b)

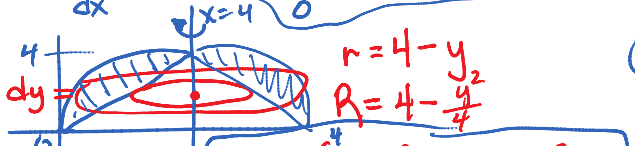


$r = \frac{y^2}{4}$

$R = y$

$\boxed{V = \pi \int_0^4 \left[(y)^2 - \left(\frac{y^2}{4}\right)^2\right] dy}$

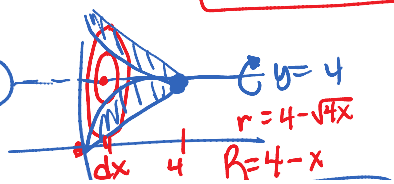
(c)



$r = 4 - y$

$R = 4 - \frac{y^2}{4}$

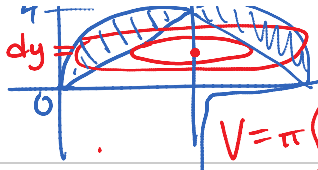
(d)



$r = 4 - \sqrt{4x}$

$R = 4 - x$

(c)



$r = 4 - \frac{y^2}{4}$

$R = 4 - y$

$$V = \pi \int_0^4 \left[\left(4 - \frac{y^2}{4}\right)^2 - (4-y)^2\right] dy$$

(d)

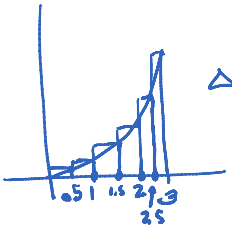
$r = 4 - \sqrt{4x}$

$R = 4 - x$

$$V = \pi \int_0^4 \left[(4-x)^2 - (4-\sqrt{4x})^2\right] dx$$

5. Approximate the area under the curve $y = x^2$ from x = 0 and x = 3 with six subintervals:

a. RRAM



$\Delta x = \frac{3}{6} = \frac{1}{2}$

b. LRAM

x	y
0	0
.5	.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9

c. MRAM

x	y
.25	.0625
.75	.5625
1.25	1.5625
1.75	3.0625
2.25	5.0625
2.75	7.5625

$A \approx RRAM_6 = .5(.25) + .5(1) + .5(2.25) + .5(4) + .5(6.25) + .5(9) = \boxed{11.375}$

$LRAM_6 = .5(0) + .5(.25) + .5(1) + .5(2.25) + .5(4) + .5(6.25) = \boxed{6.875}$

$MRAM_6 = .5(.0625) + .5(.5625) + .5(1.5625) + .5(3.0625) + .5(5.0625) + .5(7.5625) = \boxed{8.9375}$

6. Solve the initial value problem.

a. $\int \frac{dy}{dx} = \int \left(1 + x + \frac{x^2}{2}\right)$ $y(0) = 1$

$y = x + \frac{1}{2}x^2 + \frac{x^3}{6} + C$

$1 = 0 + \frac{1}{2}(0)^2 + \frac{0^3}{6} + C$

$1 = C$

$\boxed{y = x + \frac{1}{2}x^2 + \frac{x^3}{6} + 1}$

b. $\int \frac{dy}{dt} = \int \frac{1}{t+4}$, $y(-3) = 2$

$y = \ln|t+4| + C$

$2 = \ln|-3+4| + C$

$2 = C$

$\boxed{y = \ln|t+4| + 2}$

7. $f'(x) = \sin^2 x$ *and* $f(1) = 3$, *find* $f(6)$

$f(x) = F$

$\int_1^6 f'(x) = F(6) - F(1)$

(Calc)

$2.861 = F(6) - 3$

$\boxed{F(6) = 5.861}$

8. A violin made in 1785 by John Betts, one of England's finest violin makers, cost \$250 in 1924 and sold for \$7500 in 1988. Assuming a constant relative rate of appreciation, what was the rate?

$\boxed{y = y_0 e^{kt}}$

$t = 1988 - 1924$

$t = 64$

exponential

$7500 = 250 e^{64k}$

$30 = e^{64k} \longrightarrow \ln 30 = 64k$

$\boxed{k = .053}$

*you should try the slope field on pg 373 #37 and #38

$30 = e^{64k} \longrightarrow \ln 30 = 64k$

*you should try the slope field on pg 373 #37 and #38

Answers:

1. 1 2. 18 3. $\frac{\pi^2}{4}$ 4a. $\frac{32\pi}{3}$ b. $\frac{128\pi}{15}$ c. $\frac{64\pi}{5}$ d. $\frac{32\pi}{3}$

5a. 11.375 b. 6.875 c. 8.9375

6a. $y = \frac{x^3}{6} + \frac{x^2}{2} + x + 1$ b. $y = \ln(t+4) + 2$

7. 5.861 8. 5.3%

Find particular solution.

$\frac{dy}{dx} = x^2 y$ when $f(0) = 1$

$\int \frac{1}{y}\,dy = \int x^2\,dx$

$\ln|y| = \frac{1}{3}x^3 + C \longrightarrow \ln|y| = \frac{1}{3}x^3$

$\ln|1| = 0 + C$ $|y| = e^{\frac{1}{3}x^3}$

$0 = C$ $y = \pm e^{\frac{1}{3}x^3}$ $(0,1)$

$y = e^{\frac{1}{3}x^3}$