5.5 Notes - Law of Sines

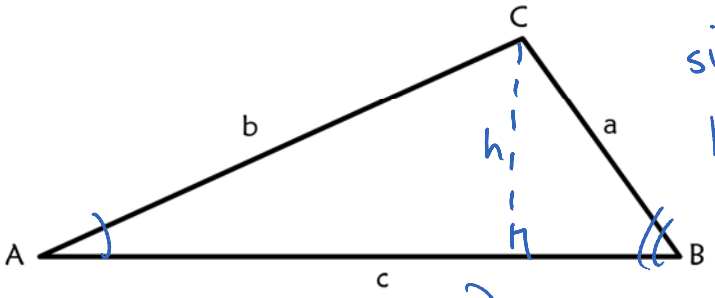
Monday, November 3, 2014
9:05 AM

Precalculus Section 5.5 – The Law of Sines

Solving a triangle means finding measures of all sides and angles.

If the triangle is a right triangle, you can use Pythagorean Theorem and/or Right Triangle Trig (SOHCAHTOA)

What if the triangle is NOT a right triangle?



(can repeat for ∠C!)

$\sin\angle A = \frac{h}{b}$ $\sin\angle B = \frac{h}{a}$

$h = b\sin\angle A$ $h = a\sin\angle B$

since both = h,

$\frac{b\sin\angle A}{b} = \frac{a\sin\angle B}{a}$

$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b}$

The Law of Sines states:

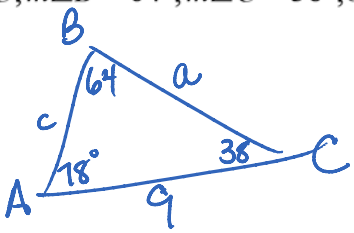
In any ΔABC with angles A, B, and C opposite sides a, b, and c, respectively, the following equation is true:

$$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$$

Examples:

1. In $\Delta ABC, m\angle B = 64°, m\angle C = 38°, b = 9$. Solve the triangle.

$m\angle A = 180° - 64° - 38° = \boxed{78°}$



$\frac{\sin 64°}{9} = \frac{\sin 78°}{a}$

$a\sin 64 = 9\sin 78$

$a = \frac{9\sin 78}{\sin 64} = \boxed{9.8}$

$\frac{\sin 64°}{9} = \frac{\sin 38°}{c}$

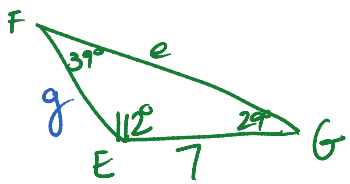
$c\sin 64 = 9\sin 38$

$c = \frac{9\sin 38}{\sin 64}$

$\boxed{c = 6.2}$

$c = 6.2$

2. In $\Delta EFG, m\angle E = 112°, m\angle G = 29°, f = 7$. Solve the triangle.



First find m∠F!

$m\angle F = 180 - 112 - 29 = \boxed{39°}$

$\frac{\sin 39°}{7} = \frac{\sin 112°}{e}$

$\boxed{e = 10.3}$

$\frac{\sin 39°}{7} = \frac{\sin 29°}{g}$

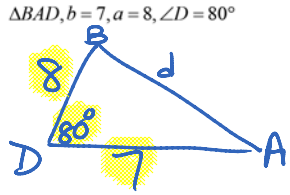
$\boxed{g = 5.4}$

Try it! Solve the triangles, using the given information.

3. $\Delta HAT, a = 6, \angle A = 36°, \angle T = 47°$

4. $\Delta TRY, \angle T = 42°, \angle Y = 98°, r = 10$

5. $\Delta BAD, b = 7, a = 8, \angle D = 80°$



$\frac{\sin 80}{d} = \frac{\sin A}{8}$

2 unknowns

➢ What pieces of information do you need to know about a triangle to use the Law of Sines?

A) AAS (2 ∠s and non-incl. side) B) ASA (2 ∠s and incl. side)

➢ When can you NOT use the Law of Sines? When given:

A) SAS (2 sides + included ∠) B) SSS (3 sides no ∠s)

C) AAA (3 ∠s, no sides)

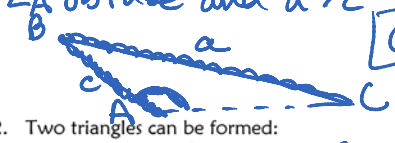
➢ When can you *maybe* use the Law of Sines? ASS (2 sides, non-included ∠)

This is called Ambiguous Case

3 Possible Outcomes when given 2 sides of a triangle and 1 non-included angle: (ASS)

1. One triangle is formed: GIVEN: ∠A, side c, and side a

∠A obtuse and a > c

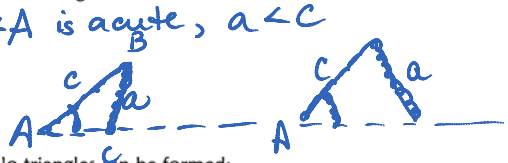


OR

∠A acute, a > c

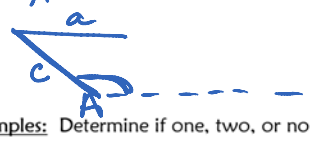
2. Two triangles can be formed:

∠A is acute, a < c



3. No triangles can be formed:

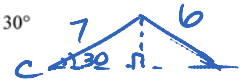
∠A obtuse c > a



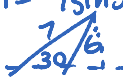
∠A acute c > a and a < h

Examples: Determine if one, two, or no triangles can be formed using the given information.

6. $\Delta ABC, b = 7, c = 6, \angle C = 30°$



$h = 7\sin 30 = 3.5$

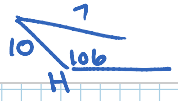


$\boxed{2\ \Delta s}$

7. $\Delta DEF, d = 7, e = 6, \angle D = 26°$

$\boxed{1\ \Delta}$

8. $\Delta GHI, h = 7, g = 10, \angle H = 106°$



$\boxed{\text{No } \Delta}$